

Qualifying Exam Syllabus

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Committee: Lior Pachter, Vera Serganova (Chair), Yun S. Song (Stats), Bernd Sturmfels.

1. MAJOR TOPIC: ALGEBRAIC COMBINATORICS (ALGEBRA)

Ordinary generating functions, the Twelvefold Way, inclusion-exclusion, exponential generating functions, Lagrange inversion, the Matrix-Tree theorem, semimodular and distributive lattices, incidence algebra, Möbius function and Möbius inversion, Eulerian posets, Euler characteristic, simplicial posets, Cohen-Macaulay posets, transfer-matrix method, symmetric functions and classical bases, Hall inner product, Schur functions, Robinson-Schensted-Knuth correspondence, Pieri formula, Jacobi-Trudi identity, Littlewood-Richardson rule.

Reference: Stanley, *Enumerative Combinatorics*, Vol. 1 (Ch 1-3, 4.7) and Vol. 2 (Ch 5, 7).

2. MAJOR TOPIC: COMPUTATIONAL BIOLOGY (APPLIED MATH)

Parametric statistical models, linear and toric models, maximum likelihood estimation, EM algorithm, implicitization of statistical models, Markov models, tree models, graphical models. Tropical arithmetic, sequence alignment, Viterbi algorithm, trees and metrics, phylogenetic trees, neighbor-joining algorithm, splits equivalence theorem, 4-point condition, cherry-picking theorem. Statistical models for mutation: MAP inference for gene-finding, rate and substitution matrices, maximum likelihood branch lengths, Jukes-Cantor model, hidden Markov models.

Reference: Pachter and Sturmfels, ed., *Algebraic Statistics for Computational Biology*, Ch 1-4.

3. MINOR TOPIC: COMMUTATIVE ALGEBRA (ALGEBRA)

Hilbert Basis theorem, localization, tensor products, Nakayama's lemma, prime avoidance, associated primes, primary decomposition, integral dependence, going-up and going-down, normal domains, flat modules, local criterion for flatness, complete local rings, Hensel's lemma, Krull dimension, Noether normalization, Hilbert's Nullstellensatz, monomial orderings, division algorithm, Gröbner bases, Buchberger's algorithm.

Reference: Eisenbud, *Commutative Algebra with a View Toward Algebraic Geometry*, Ch 2-4, 6-8, 13.1, 15.1-4.