**Math 1B Section 101**
**02 Sep 2009 Quiz #1**

Name: ______________________________ Score: /10

SID#: ______________________________ Time: 15 mins

1. (4 points) Evaluate $\int_4^9 \frac{\ln y}{\sqrt{y}} dy$. Give your answer in the form $a \ln 3 + b \ln 2 + c$.

**Ans 1**:

$$\begin{array}{ll} u = \ln y & dv = y^{-1/2} dy \\ du = \frac{1}{y} dy & v = 2y^{1/2} \end{array}$$

$$\begin{aligned} \int_4^9 \frac{\ln y}{\sqrt{y}} dy &= 2\Big[\sqrt{y}\ln y\Big]_4^9 - 2\int_4^9 \frac{1}{\sqrt{y}} dy \\ &= 2\Big[\sqrt{y}\ln y\Big]_4^9 - 4\Big[\sqrt{y}\Big]_4^9 \\ &= 2(\sqrt{9}\ln 9 - \sqrt{4}\ln 4) - 4(\sqrt{9} - \sqrt{4}) \\ &= 12\ln 3 - 8\ln 2 - 4 \end{aligned}$$

**Ans 2**: Integration by substitution. Let $u = \sqrt{y}$. Then, $du = \frac{1}{2\sqrt{y}} dy$.

$$\begin{aligned} \int_4^9 \frac{\ln y}{\sqrt{y}} dy &= \int_4^3 2\ln u^2 du \\ &= 4\Big[u \ln u - u\Big]_2^3 \\ &= 12\ln 3 - 8\ln 2 - 4 \end{aligned}$$

2. (6 points) Find $\int e^{2\theta} \sin 3\theta d\theta$.
(Hint: Do integration-by-parts twice, and rearrange the resulting equation.)

**Ans 1**:

$$\begin{array}{ll} u = e^{2\theta} & dv = \sin 3\theta d\theta \\ du = 2e^{2\theta} d\theta & v = -\frac{1}{3}\cos 3\theta \end{array}$$

$$\int e^{2\theta} \sin 3\theta d\theta = -\frac{1}{3} e^{2\theta} \cos 3\theta + \frac{2}{3} \int e^{2\theta} \cos 3\theta d\theta.$$

$$\begin{array}{ll} u = e^{2\theta} & dv = \cos 3\theta d\theta \\ du = 2e^{2\theta} d\theta & v = \frac{1}{3}\sin 3\theta \end{array}$$

$$\int e^{2\theta} \sin 3\theta d\theta = -\frac{1}{3} e^{2\theta} \cos 3\theta + \frac{2}{3}(\frac{1}{3} e^{2\theta} \sin 3\theta - \frac{2}{3} \int e^{2\theta} \sin 3\theta d\theta)$$

Rearranging the above equation, we get

$$\frac{13}{9} \int e^{2\theta} \sin 3\theta d\theta = -\frac{1}{3} e^{2\theta} \cos 3\theta + \frac{2}{9} e^{2\theta} \sin 3\theta + C$$

Therefore,

$$\int e^{2\theta} \sin 3\theta d\theta = -\frac{3}{13} e^{2\theta} \cos 3\theta + \frac{2}{13} e^{2\theta} \sin 3\theta + C$$

**Ans 2**:

$$\begin{array}{ll} u = \sin 3\theta & dv = e^{2\theta} d\theta \\ du = 3\cos 3\theta d\theta & v = \frac{1}{2} e^{2\theta} \end{array}$$

$$\int e^{2\theta} \sin 3\theta d\theta = \frac{1}{2} e^{2\theta} \sin 3\theta - \frac{3}{2} \int e^{2\theta} \cos 3\theta d\theta.$$

$$\begin{array}{ll} u = \cos 3\theta & dv = e^{2\theta} d\theta \\ du = -3\sin 3\theta d\theta & v = \frac{1}{2} e^{2\theta} \end{array}$$

$$\int e^{2\theta} \sin 3\theta d\theta = \frac{1}{2} e^{2\theta} \sin 3\theta - \frac{3}{2}(\frac{1}{2} e^{2\theta} \cos 3\theta + \frac{3}{2} \int e^{2\theta} \sin 3\theta d\theta)$$

Rearranging the above equation, we get

$$\frac{13}{4} \int e^{2\theta} \sin 3\theta d\theta = -\frac{3}{4} e^{2\theta} \cos 3\theta + \frac{1}{2} e^{2\theta} \sin 3\theta + C$$

Therefore,

$$\int e^{2\theta} \sin 3\theta d\theta = -\frac{3}{13} e^{2\theta} \cos 3\theta + \frac{2}{13} e^{2\theta} \sin 3\theta + C$$

3. (bonus, 0 points) Prove $\int x^n e^x dx = (-1)^n n! \left(1 - \frac{x}{1!} + \frac{x^2}{2!} - \cdots + (-1)^n \frac{x^n}{n!}\right) e^x + C.$

**Ans**:

$$\begin{array}{ll} u = x^n & dv = e^x dx \\ du = nx^{n-1}dx & v = e^x \end{array}$$

$$\begin{aligned}
\int x^n e^x dx &= x^n e^x - n \int x^{n-1} e^x dx \\
&= x^n e^x - n(x^{n-1}e^x - (n-1)\int x^{n-2} e^x dx) \\
&= x^n e^x - nx^{n-1}e^x + n(n-1)\int x^{n-2} e^x dx \\
&= \cdots \\
&= x^n e^x - nx^{n-1}e^x + n(n-1)x^{n-2}e^x - \cdots + (-1)^n n(n-1)\cdots(2)(1)\int x^0 e^x dx \\
&= x^n e^x - nx^{n-1}e^x + n(n-1)x^{n-2}e^x - \cdots + (-1)^n n! e^x + C \\
&= (-1)^n n! \left(1 - \frac{x}{1!} + \frac{x^2}{2!} - \cdots + (-1)^n \frac{x^n}{n!}\right) e^x + C
\end{aligned}$$

[End of Quiz]

## Quiz Statistics

| Score | Count |
|---|---|
| 0 | 0 |
| 1 | 0 |
| 2 | 1 |
| 3 | 0 |
| 4 | 2 |
| 5 | 0 |
| 6 | 3 |
| 7 | 4 |
| 8 | 5 |
| 9 | 8 |
| 10 | 6 |

Ave = 7.86