Important Derivatives	Important Integrals
$\frac{d}{dx}x^n = nx^{n-1}$	$(*)\ \int \frac{1}{ax+b}\ dx = \frac{1}{a}\ln\lvert ax+b\rvert + C$
$\frac{d}{dx}\ln x = \frac{1}{x}$	$(*)\ \int \frac{1}{x^2+a^2}\ dx = \frac{1}{a}\tan^{-1}\frac{x}{a} + C$
$\frac{d}{dx}e^x = e^x$	$(*)\ \int \frac{x}{x^2+a^2}\ dx = \frac{1}{2}\ln\lvert x^2+a^2\rvert + C$
$\frac{d}{dx}\sin x = \cos x$	$\int \frac{1}{x^2-a^2}\ dx = \frac{1}{2a}\ln\lvert\frac{x-a}{x+a}\rvert + C$
$\frac{d}{dx}\cos x = -\sin x$	$\int \frac{1}{\sqrt{a^2-x^2}}\ dx = \sin^{-1}\frac{x}{a} + C$
$\frac{d}{dx}\tan x = \sec^2 x$	$\int \frac{1}{\sqrt{x^2\pm a^2}}\ dx = \ln\lvert x+\sqrt{x^2\pm a^2}\rvert + C$
$\frac{d}{dx}\sec x = \sec x\tan x$	$\int \tan x\ dx = \ln\lvert\sec x\rvert + C$
$\frac{d}{dx}\sin^{-1}x = \frac{1}{\sqrt{1-x^2}}$	$\int \sec x\ dx = \ln\lvert\sec x + \tan x\rvert + C$
$\frac{d}{dx}\cos^{-1}x = -\frac{1}{\sqrt{1-x^2}}$	$\int \sec^2 x\ dx = \tan x + C$
$\frac{d}{dx}\tan^{-1}x = \frac{1}{1+x^2}$	$\int \sec x\tan x\ dx = \sec x + C$
	$\int \ln x\ dx = x\ln x - x + C$

Differentiate/Integrate a^x
Write a^x as $e^{(\ln a)x}$

Pythagorean Identities
$\sin^2 x + \cos^2 x = 1$
$\tan^2 x + 1 = \sec^2 x$
$1 + \cot^2 x = \csc^2 x$

Half-angle Identities
$\cos^2 x = \frac{1+\cos 2x}{2}$
$\sin^2 x = \frac{1-\cos 2x}{2}$
$\sin x\cos x = \frac{\sin 2x}{2}$

Trig Substitutions:

Expression	$x\sqrt{\pm x^2 \pm a^2}$	$\sqrt{a^2-x^2}$	$\sqrt{x^2+a^2}$	$\sqrt{x^2-a^2}$
Identity		$1-\sin^2 x = \cos^2 x$	$\tan^2 x + 1 = \sec^2 x$	$\sec^2 x - 1 = \tan^2 x$
Substitution	$u = \pm x^2 \pm a^2$	$x = \sin\theta$	$x = \tan\theta$	$x = \sec\theta$

Partial Fractions:

Factor	Partial Fractions	Integral
$(ax+b)^k$	$\frac{A_1}{ax+b} + \frac{A_2}{(ax+b)^2} + \cdots + \frac{A_k}{(ax+b)^k}$	substitute $u = ax+b$, use $(*)$
(ax^2+bx+c)	$\frac{Ax+B}{ax^2+bx+c}$	complete the square, use $(*)$

Mantra for $\int \sin^m x \cos^n x \ dx$:
If power of $\sin x$ is positive odd, pull out a $\sin x$.
If power of $\cos x$ is positive odd, pull out a $\cos x$.
Otherwise, seek half angels for help.

Mantra for $\int \tan^m x \sec^n x \ dx$:
If power of $\sec x$ is positive even, pull out a $\sec^2 x$.
If power of $\tan x$ is positive odd, pull out a $\sec x \tan x$.
Otherwise, convert all $\tan\theta$ to $\sec\theta$. Go to the start!
Only the odd pure $\sec\theta$ is stubborn. Kill him by parts!

Identities for $\int \sin mx \cos nx \ dx$: (given in exam)
$\sin A \cos B = \frac{1}{2}[\sin(A-B) + \sin(A+B)]$
$\sin A \sin B = \frac{1}{2}[\cos(A-B) - \cos(A+B)]$
$\cos A \cos B = \frac{1}{2}[\cos(A-B) + \cos(A+B)]$

Approximate Integration $\int_a^b f(x) \ dx$:

Midpoint	$M_n = \Delta x[f(\bar{x}_1) + f(\bar{x}_2) + \cdots + f(\bar{x}_n)], \quad \bar{x}_i = \dfrac{x_{i-1} + x_i}{2}$
Trapezium	$T_n = \dfrac{\Delta x}{2}[f(x_0) + 2f(x_1) + \cdots + 2f(x_{n-1}) + f(x_n)]$
Simpson's	$S_n = \dfrac{\Delta x}{3}[f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + \cdots + 4f(x_{n-1}) + f(x_n)]$

Error Bounds for Approximate Integration:

Given in exam	Memorize this!
$\lvert E_M \rvert \leq \dfrac{K(b-a)^3}{24n^2}$	$\lvert f^{(2)}(x) \rvert \leq K$ for $a \leq x \leq b$
$\lvert E_T \rvert \leq \dfrac{K(b-a)^3}{12n^2}$	$\lvert f^{(2)}(x) \rvert \leq K$ for $a \leq x \leq b$
$\lvert E_S \rvert \leq \dfrac{K(b-a)^5}{180n^4}$	$\lvert f^{(4)}(x) \rvert \leq K$ for $a \leq x \leq b$

p-Test for Improper Integrals:

$\displaystyle\int_1^\infty \frac{1}{x^p} \ dx$ converges for $p > 1$, diverges otherwise.
$\displaystyle\int_0^1 \frac{1}{x^p} \ dx$ converges for $p < 1$, diverges otherwise.

Standard Comparison Test for Improper Integrals:

If $0 \leq g(x) \leq f(x)$ for x in $[a, \infty)$, then

$$\int_a^\infty f(x)\ dx \text{ converges} \Rightarrow \int_a^\infty g(x)\ dx \text{ converges.}$$

$$\int_a^\infty g(x)\ dx \text{ diverges} \Rightarrow \int_a^\infty f(x)\ dx \text{ diverges.}$$

(Similarly for other improper integrals of type 2 on $[a, b)$ and $(a, b]$.)

Arc Length, Surface Area:

$ds = \sqrt{1 + (\frac{dy}{dx})^2}\ dx = \sqrt{1 + (\frac{dx}{dy})^2}\ dy$

Arc Length $= \int\ ds$

Area of Surface of Revolution (about x-axis) $= \int 2\pi y\ ds$

Area of Surface of Revolution (about y-axis) $= \int 2\pi x\ ds$

Hydrostatic Pressure:

Pressure $= \rho g d$, Force $=$ Pressure $\times$ Area

Consider a submerged vertical plate whose width at depth x is $f(x)$.

Then, the hydrostatic force on the plate for $a \leq x \leq b$ is $\rho g \int_a^b x f(x)\ dx$.

Moments, Centroid:

Moment about x-axis $= M_x = \rho \int_a^b \frac{1}{2} f(x)^2\ dx$

Moment about y-axis $= M_y = \rho \int_a^b x f(x)\ dx$

Centroid $= (\bar{x}, \bar{y}) = (\frac{M_y}{\text{Mass}}, \frac{M_x}{\text{Mass}}) = (\frac{\int_a^b x f(x)\ dx}{\text{Area}}, \frac{\int_a^b \frac{1}{2} f(x)^2\ dx}{\text{Area}})$

If region is symmetrical about the y-axis, then $\bar{x} = 0$. (Similarly for $\bar{y}$.)

Pappus' Theorem. The volume of a solid of rotation of a region is the product of the area of the region with the distance travelled by the centroid during the rotation.

Evaluation of limits of sequences

i. Sum, difference, product, quotient, scaling, taking powers

ii. If $\lim f(x) = L$ and $f(n) = a_n$, then $\lim a_n = L$.

iii. If $\lim a_n = L$ and f is continuous at L, then $\lim f(a_n) = f(L)$.

iv. If $\lim |a_n| = 0$, then $\lim a_n = 0$.

v. Squeeze theorem

vi. Monotone convergent theorem

Geometric sequences

i. $\{r^n\}$ is convergent for $-1 < r \leq 1$, divergent otherwise.

ii. $\lim r^n = 0$ for $|r| < 1$ and $\lim r^n = 1$ for $r = 1$.

Evaluation of series

i. If $\sum a_n$ is convergent, then $\lim a_n = 0$.
If $\lim a_n$ does not exist, or $\lim a_n \neq 0$, then $\sum a_n$ is divergent.

ii. If $\sum a_n$, $\sum b_n$ are convergent, so are $\sum ca_n$, $\sum(a_n + b_n)$ and $\sum(a_n - b_n)$.

Geometric series

i. $\sum ar^{n-1}$ convergent if $|r| < 1$, divergent otherwise.

ii. $\sum_{n=1}^{\infty} ar^{n-1} = a/(1-r)$ for $|r| < 1$.

The Integral Test and Estimates of Sums

$a_n = f(n)$, f continuous, positive, decreasing on $[1, \infty)$.

a. Integral test: $\sum a_n$ convergent $\Leftrightarrow \int_1^{\infty} f(x)\, dx$ convergent

b. Remainder estimate: $\int_{n+1}^{\infty} f(x)\, dx \leq s - s_n \leq \int_n^{\infty} f(x)\, dx$

c. Bounds on sum: $s_n + \int_{n+1}^{\infty} f(x)\, dx \leq s \leq s_n + \int_n^{\infty} f(x)\, dx$

d. p-Test: $\sum 1/n^p$ convergent for $p > 1$, divergent otherwise.